Childhood Skill Formation and Intergenerational Mobility Trends

January 5, 2023

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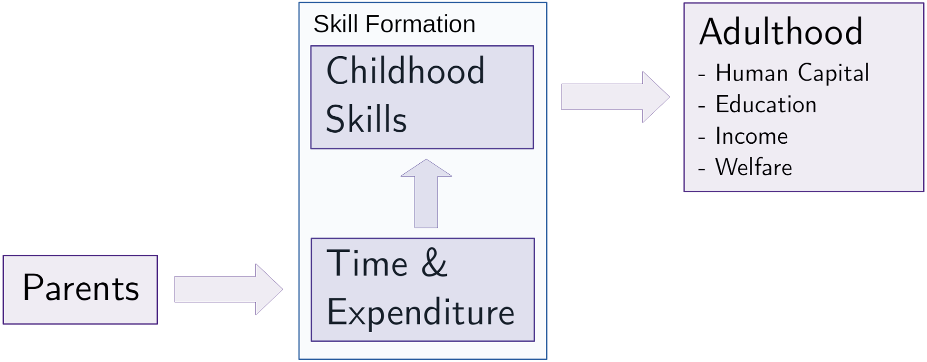
What is Childhood Skill Formation? Why is it Important?

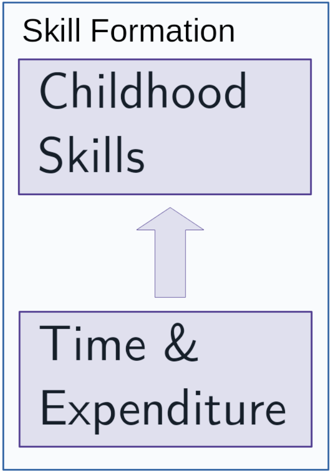
- Skill formation starts during childhood in the family.
- Long-term effects in adulthood, such as human capital, education, income.

What is Childhood Skill Formation? Why is it Important?

- Parents invest time and expenditure for children to produce skill.
  - Time: Games, learning and leisure activities.
  - Expenditure: Toys, books, extracurricular activities.



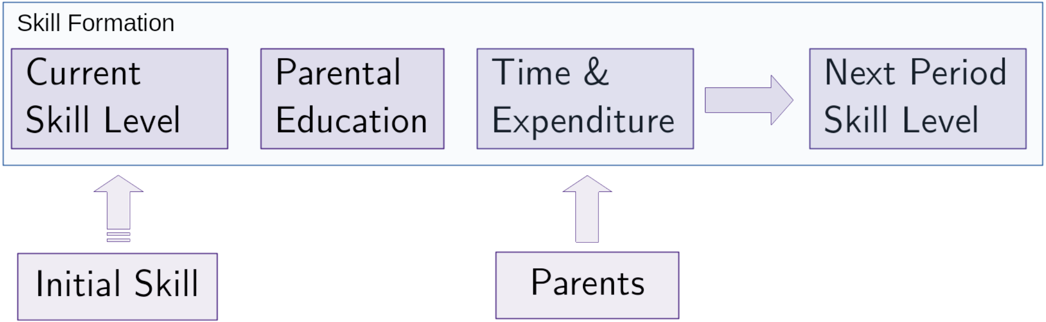
A Flexible Estimation: New Results, Significant Implications



This paper:

- Flexible estimation of childhood skill formation.
- No restrictive functional form assumptions.
- New results with significant implications.

Childhood Skill Formation Function



Result 1: Investment is more productive for low-skilled children

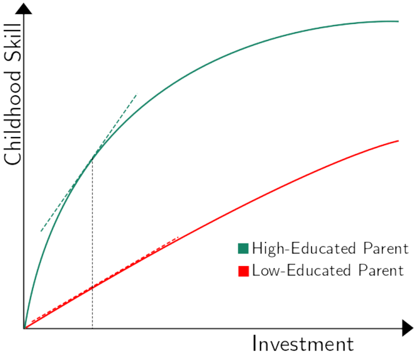
- The investment is more productive if the child currently has a low skill level.
- Current skill level can be low because of missing investments at earlier ages.

Implications

- Possible to recover missing investment at earlier ages by investing now.
- Policy can create a larger impact by focusing on low-skilled children,
- Not necessarily only at early ages, which currently capture a lot of attention.

Result 2: Investment is more productive for more educated parents

- The investment is more productive if parents are more educated.
- Return to the investment, measured in elasticity, is decreasing,
  - At a faster rate for more educated parents (more concavity).
  - They hit the flat region more quickly than low-educated parents.



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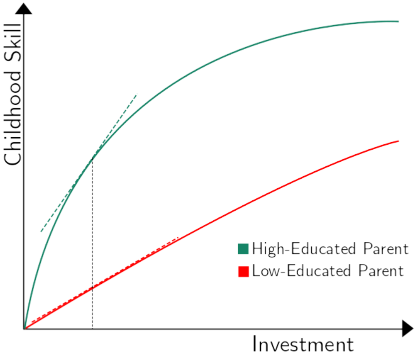
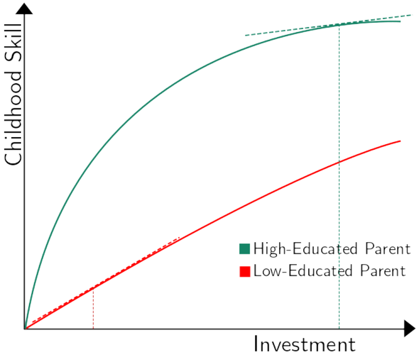
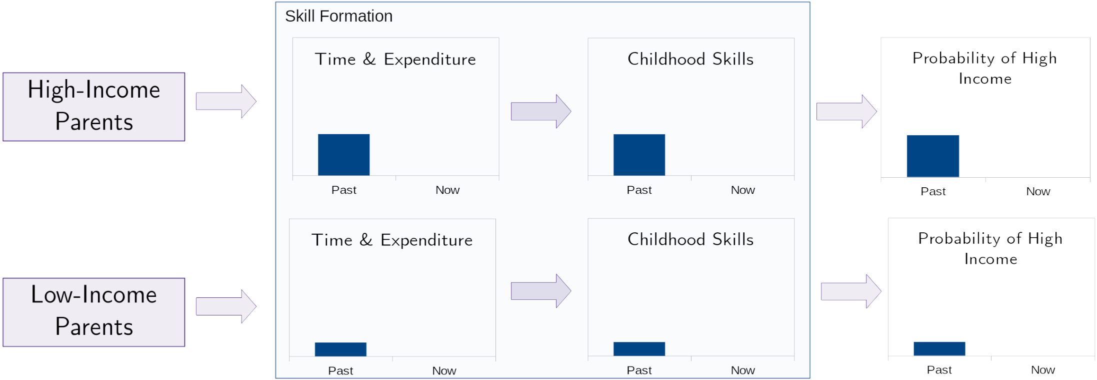
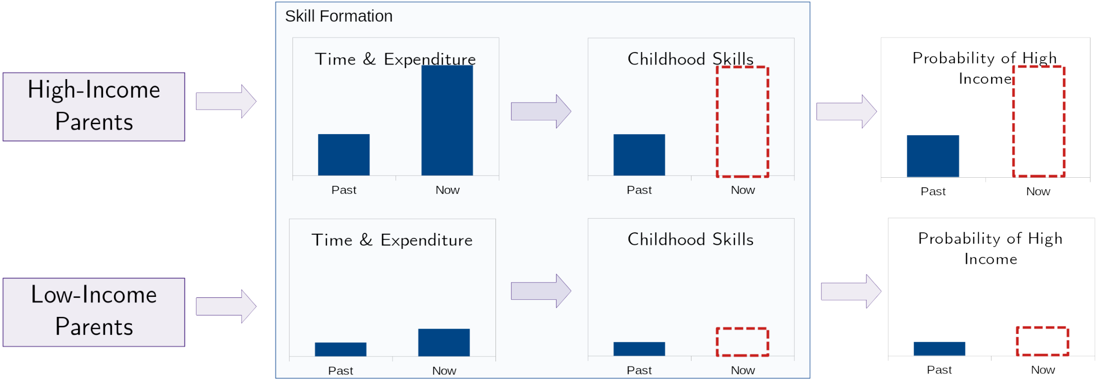




Inequality in investment is rising, implications for mobility?



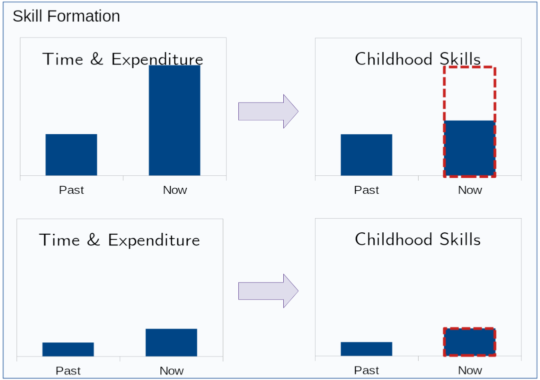
Inequality in investment is rising, implications for mobility?

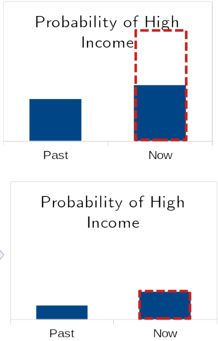


Rising inequality may not lead lower mobility across generations.

High-Income Parents

Low-Income Parents





How and Why different from the literature: CES case

- The literature typically uses the CES production function.
- Limited because it forces one of the following two cases:

Inputs are either Complements

- Investment is more productive for
  - more skilled children,
  - more educated parents.
- Return (in elasticity) is decreasing.

or Substitutes

- Investment is more productive for
  - less skilled children,
  - less educated parents.
- Return (in elasticity) is increasing.

How and Why different from the literature: CES case

- The literature typically uses CES production function.
- Limited because CES forces one of the following two cases.
- My results suggest that it is restrictive.

Inputs are either Complements

- Investment is more productive for
  - more skilled children,
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- Return (in elasticity) is decreasing.

or Substitutes

- Investment is more productive for
  - less skilled children,
  - less educated parents.
- Return (in elasticity) is increasing.

Literature

Estimation of Childhood Skill Formation Function

Cunha, Heckman and Schennach (2010) ECTA, Agostinelli and Wiswall (fort.) JPE, Attanasio et. el. (2020) AER, Cunha and Heckman (2007, 2008), Del Boca et al. (2014), Heckman and Mosso (2014), Cunha et al. (2021)

Inequality and Intergenerational Mobility

Becker et. al. 2018 JPE, Becker and Tomes (1979, 1986), Loury (1981), Solon (2004)

Intergenerational Mobility Trends

Davis and Mazumder (2022), Song et. al. (2019), Chetty et. al. (2014), Corak (2013), Solon and Lee (2009), Aaronson and Mazumder (2008)

Outline of The Talk

Data: PSID - CDS

Panel Study of Income Dynamics - Child Development Supplement

- Three periods 1997, 2002, 2007.
- Parental Investment: Time and Expenditure
  - 24 hours diaries of children: Games, quality time, educational activities.
  - Child-related expenditures: Books, toys, extracurricular activities.
- Noisy Childhood Skill Measures: Standardized cognitive tests.
  - Letter-Word Identification, Applied Math Problems and Passage Completion.
- Income, years of education, household composition.

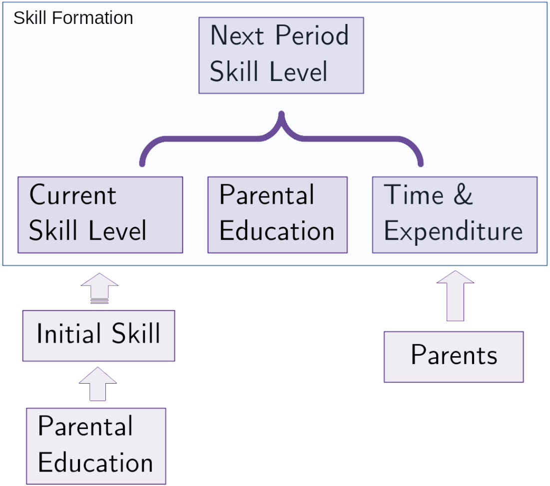
Empirical Model: Bird's-Eye View

Initial Skill

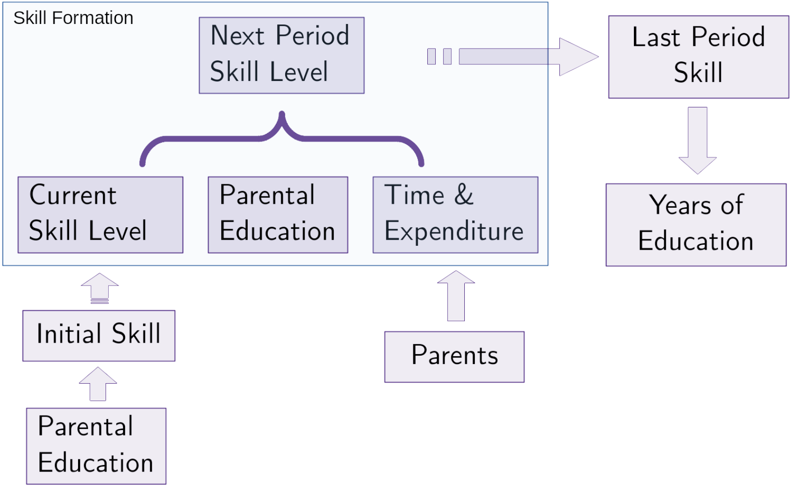
↑

Parental Education

Empirical Model: Bird's-Eye View



Empirical Model: Bird's-Eye View



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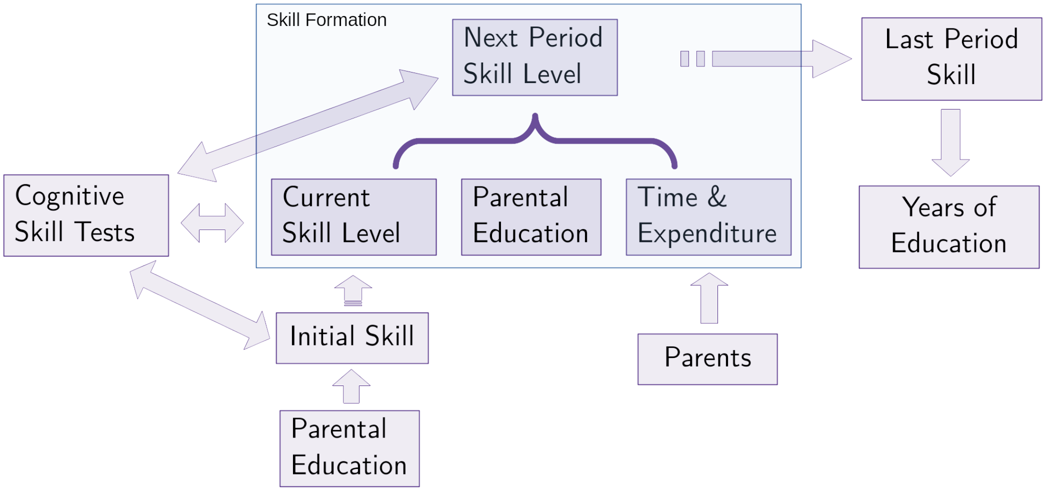

Childhood Skill Formation Function

Let θ_j be child's skill level at period j,

$$\theta_{j+1} = F(\theta_j, \theta_P, I_j, u_j), \quad u_j \sim U[0,1].$$

Inputs:

- Current Skill Level: θ_j
- Parental Education: $\theta_P = g(\theta_{father}, \theta_{mother})$
- Investment: $I_j = h(t_j^{father}, t_j^{mother}, m_j)$

Childhood Skill Formation Function

Let θ_j be child's skill level at period j,

$$\theta_{j+1} = F(\theta_j, \theta_P, I_j, u_j), \quad u_j \sim U[0,1].$$

CES with Normal Noise

$$\theta_{j+1} = \left[\gamma_\theta \theta_j^\phi + \gamma_P \theta_P^\phi + (1 - \gamma_\theta - \gamma_P) I_j^\phi\right]^{\frac{1}{\phi}} exp(\varepsilon_j), \quad \varepsilon_j \sim \mathcal{N}(0, \sigma_\varepsilon)$$

Childhood Skill Formation Function: Parametrize

Approximate the unknown skill formation function,

$$F(\theta_j, I_j, \theta_P, u_j) = \sum_{k=0}^{K} a_k(u_j)\varphi_k(\theta_j, I_j, \theta_P), \quad u_j \sim U[0,1]$$

where,

- φ_k's orthogonal polynomials,
- $a_k(u_j)$ is polynomial coefficients.
- Estimate for a grid $\{u^0, u^1, \ldots, u^L\}$ with quantile regressions.

Childhood Skill Formation Function: Parametrize

$$
\begin{aligned}
F(\theta_j, I_j, \theta_P, u_j) &= \sum_{k=0}^{K} a_k(u_j)\varphi_k(\theta_j, I_j, \theta_P), \quad u_j \sim U[0,1] \\
&= a_0(u_j) + a_1(u_j)\theta_j + a_2(u_j)I_j + a_3(u_j)\theta_P + a_4(u_j)\theta_j I_j + a_5(u_j)\theta_j I_j^2 \ldots
\end{aligned}
$$

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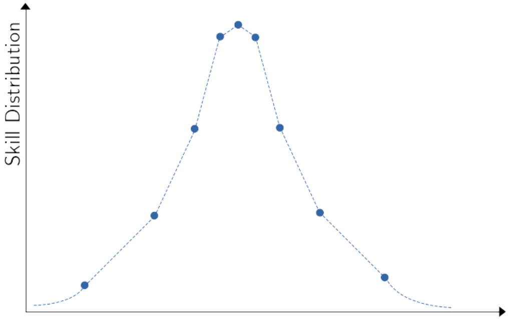


Moment conditions to minimize

- Each part of the empirical model has a moment condition.
- Let $R(\theta, X, \tilde{\Lambda})$ denote all moment conditions and true parameter values solves,

$$\Lambda = \arg\min_{\tilde{\Lambda}} \mathbb{E}\left[R(\theta, X, \tilde{\Lambda})\right]$$

where,

- θ unobservable childhood skills,
- X all the data,
- Λ all parameters.

Moment conditions to minimize

- Each part of the empirical model has a moment condition.
- Let $R(\theta, X, \tilde{\Lambda})$ denote all moment conditions and true parameter values solves,

$$\Lambda = \arg\min_{\tilde{\Lambda}} \mathbb{E}\left[R(\theta, X, \tilde{\Lambda})\right]$$

- Its sample counter part is not feasible because skills are not observable,

$$\hat{\Lambda} = \arg\min_{\tilde{\Lambda}} \frac{1}{N} \sum_{i=0}^{N} R(\theta_i, X_i, \tilde{\Lambda}).$$

EM Algorithm

Guess parameter values, $\hat{\Lambda}^{(0)}$ and start iteration with $s = 0$.

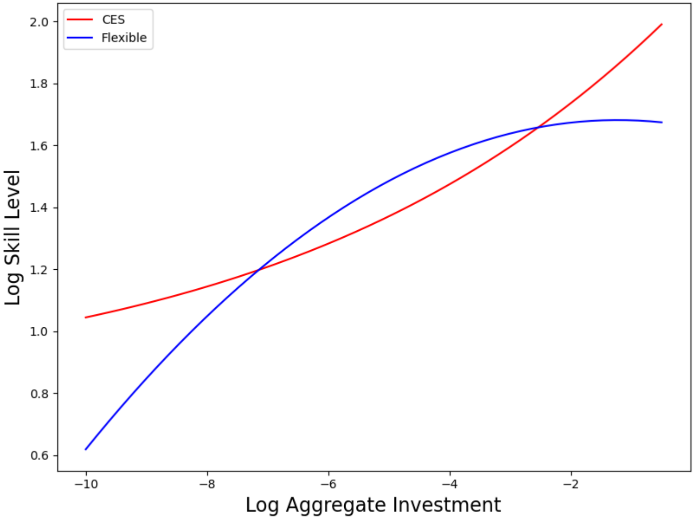
- Simulate underlying skill levels for each child i,

$$\theta_i^{(s)} \sim f(\theta \mid X_i, \hat{\Lambda}^{(s)}) \quad \text{for} \quad i = 0, \ldots, N,$$

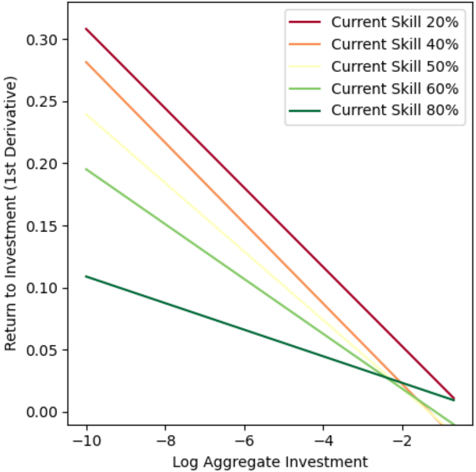
- Solve sample counterpart of minimization and update the parameters,

$$\hat{\Lambda}^{s+1} = \arg\min_{\tilde{\Lambda}} \frac{1}{N} \sum_{i=0}^{N} R(\theta_i^{(s)}, X_i, \tilde{\Lambda}).$$

Returns are decreasing and almost flat at top



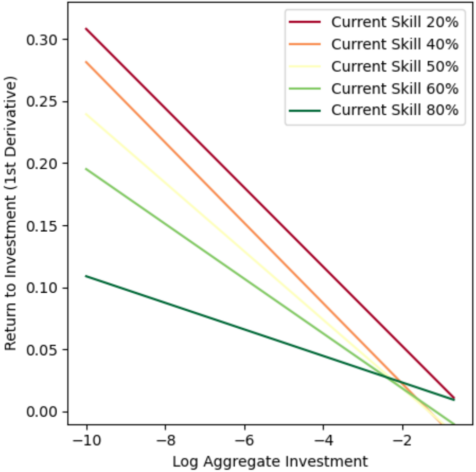
Current Skill Level and Investment are Substitutes



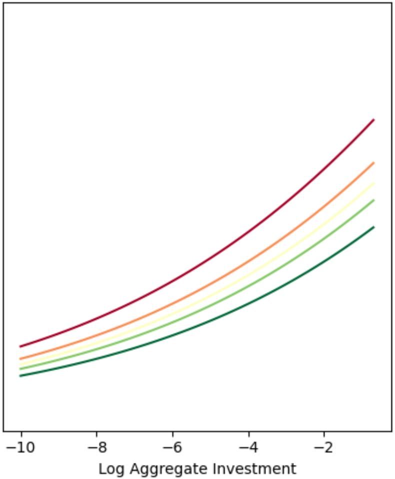
$$\mathbb{E}\left[\frac{\partial^2 \ln F(I_t, \theta_t, \theta_P, u_t)}{\partial \ln I_t \partial \ln \theta_t}\right] < 0$$

Current Skill Level and Investment are Substitutes

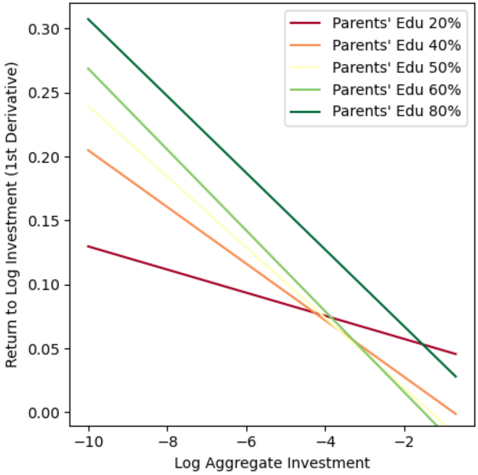
Flexible



CES with Normal Distribution



Parents' Education and Investment are Complements

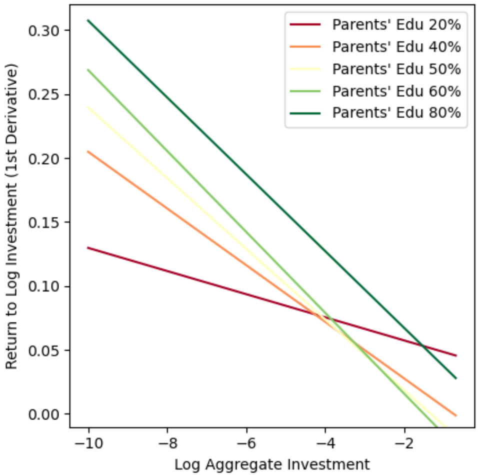


$$\mathbb{E}\left[\frac{\partial^2 \ln F\left(I_t, \theta_t, \theta_P, u_t\right)}{\partial \ln I_t \partial \ln \theta_P}\right] > 0$$

$$\mathbb{E}\left[\frac{\partial^3 \ln F\left(I_t, \theta_t, \theta_P, u_t\right)}{\partial^2 \ln I_t \partial \ln \theta_P}\right] < 0$$

Parents' Education and Investment are Complements

Flexible



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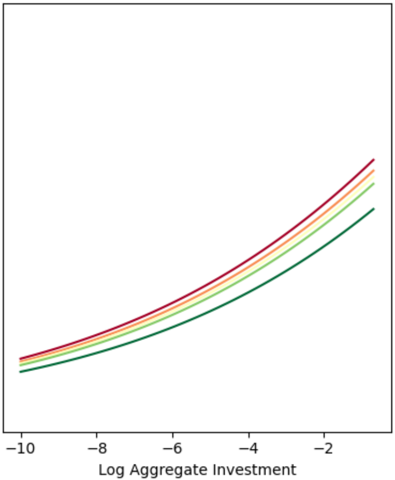
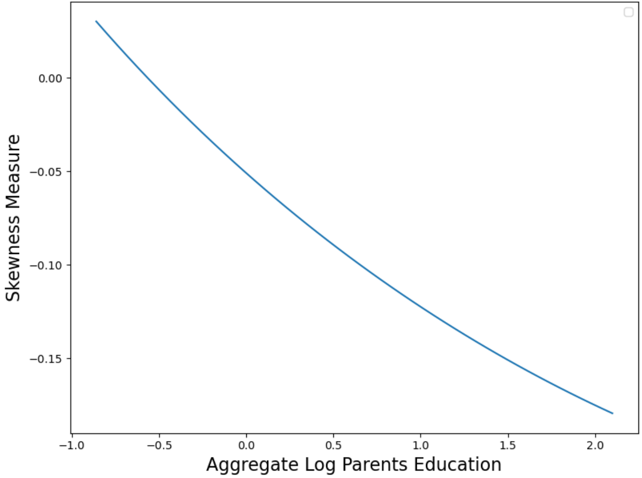
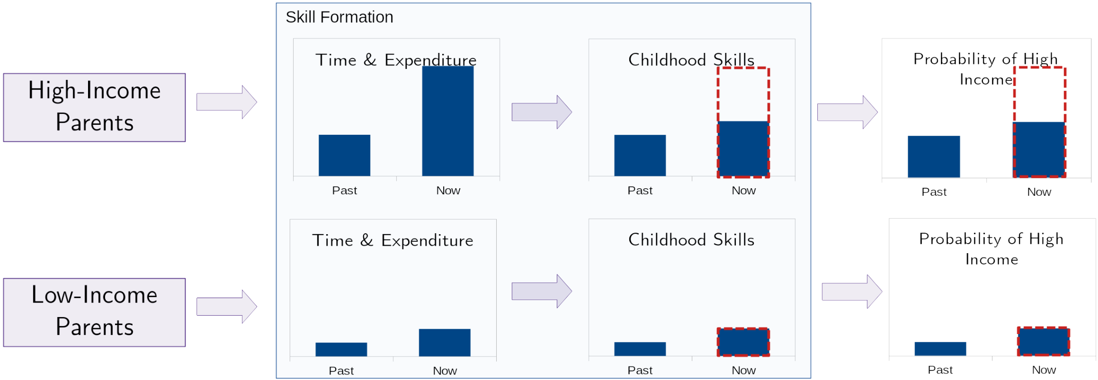


More Negative Skewness for More Educated Parents

Children of high-educated parents are more subject to negative risk



Rising inequality may not lead lower mobility across generations.

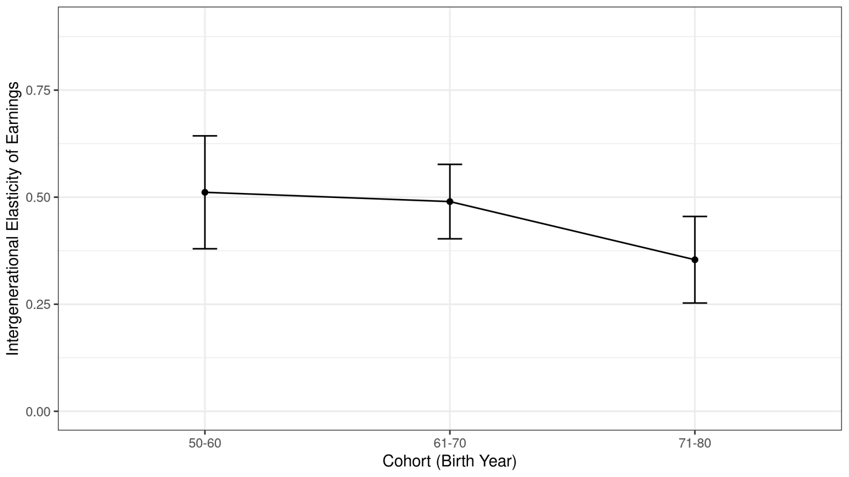


Estimate a trend for mobility in earnings across generations

$$\ln y_{ic}^{child} = \alpha_c + \beta_c \ln y_{ic}^{parent}$$

- y_{ic} is approximated by average earnings over ages around 40.
    - As in Mazumder (2016).
- Group cohorts in 10 years in PSID.

Intergenerational earning mobility trend seems flat



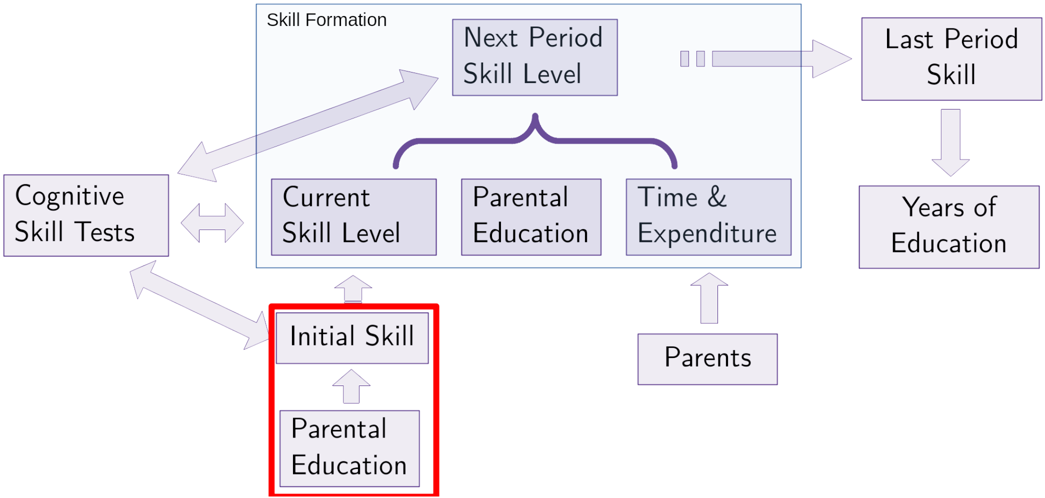
Conclusion

- I provide a flexible estimation for childhood skill formation function.
- No restrictive functional or distributional form assumptions.

- Investment is more productive for currently low-skilled children.
- Policy interventions should focus on disadvantaged children even at later ages.

- Returns are low for high-income parents and high for low-income parents.
- Rising inequality may not lead to lower mobility.

Empirical Model: Bird's-Eye View



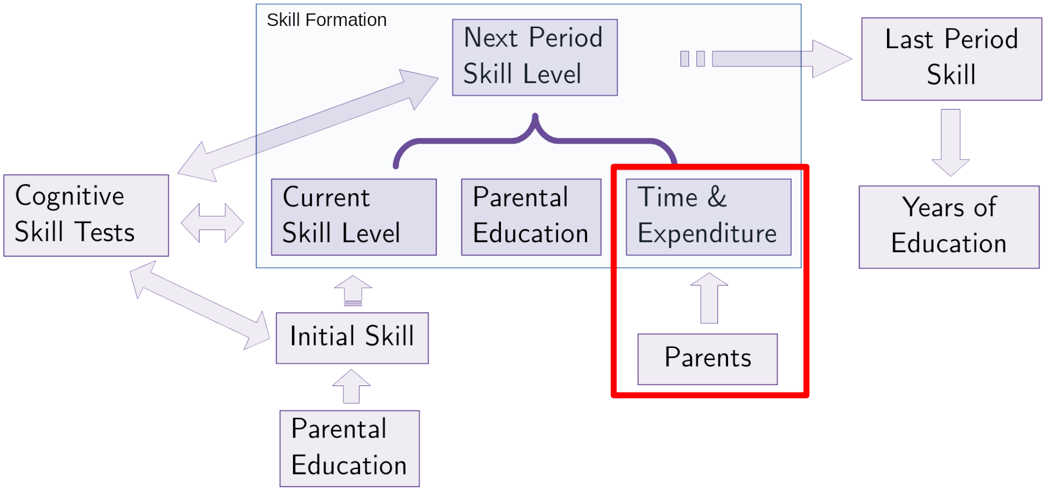
Initial Childhood Skill Distribution

Let initial level of skills depend on age and parents' education,

$$\theta_0 = F_0(age_0, \theta_P, u_0), \quad u_0 \sim U[0,1],$$

with $u_0 \sim U[0,1]$.

Empirical Model: Bird's-Eye View



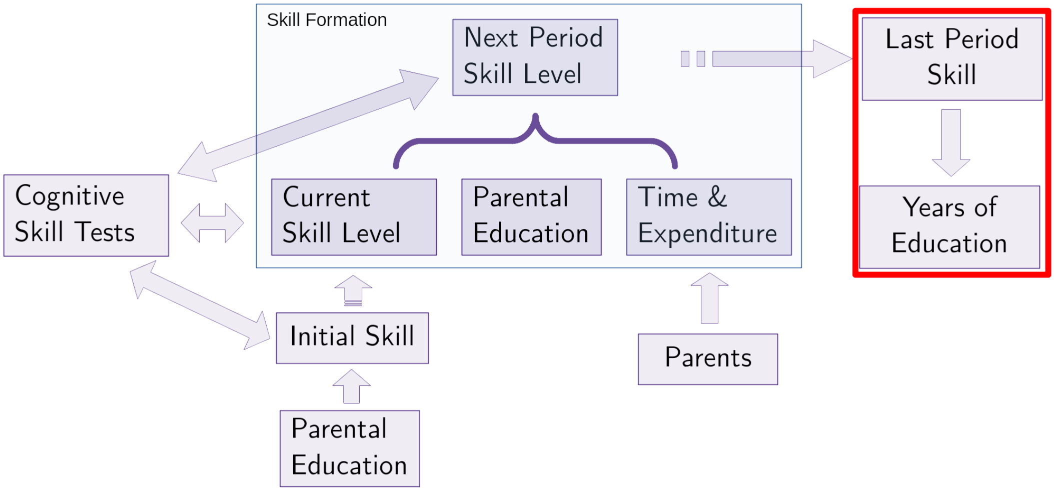
Parental Investment Function

Parental investment, time (t_j^k) and expenditure (m_j), functions are given by,

$$m_j = M(\theta_j, \theta_P, y_j, u_j^M),$$
$$t_j^k = T^k(\theta_j, \theta_P, y_j, u_{kj}^T) \quad \text{for} \quad k = father, mother,$$

where y_j is total household income and $u_j^M, u_{kj}^T \sim U[0,1]$.

Empirical Model: Bird's-Eye View



Last Period Skill to Years Of Education

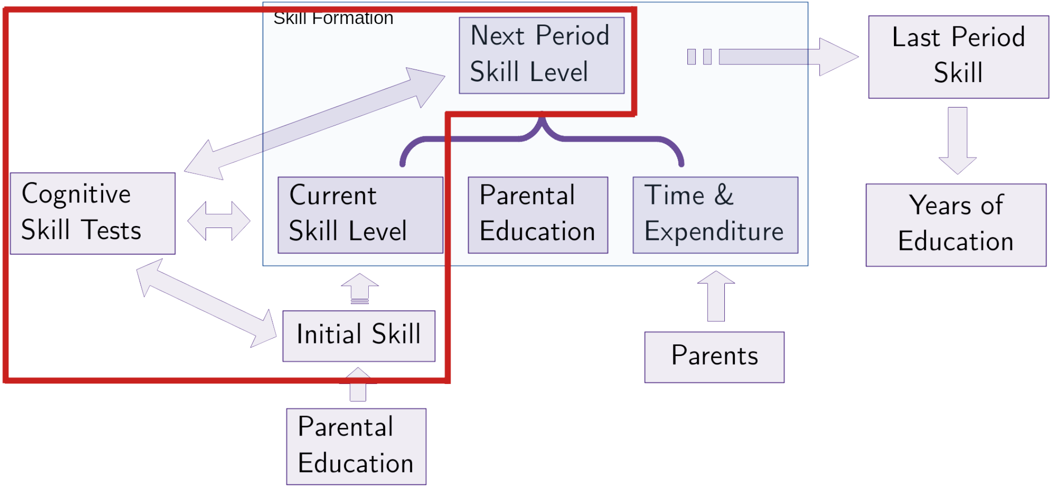
- Binomial distribution for final years of education.

$$EduYears \sim Binomial(n, p).$$

- $n = 17$ is max years of education.
- p is the probability parameter given by,

$$p = \Lambda(\theta_T, \theta_P, age_T).$$

Empirical Model: Bird's-Eye View



Childhood Skill Measurement: Cognitive Tests

For child with skill level θ, probability of answering a question i in test t correct:

$$Prob_{ti} = \frac{\exp\left(\alpha_t + \beta_t \theta - d_i\right)}{1 + \exp\left(\alpha_t + \beta_t \theta - d_i\right)}.$$

- Normalize one test as $\alpha_t = 0, \beta_t = 1$.

Moment conditions to minimize

- Each part of the empirical model has a moment condition.
- Let $R(\Theta, X, \tilde{\Lambda})$ denote all moment conditions and true parameter values solves,

$$\Lambda = \arg\min_{\tilde{\Lambda}} \mathbb{E}_X \left[\mathbb{E}_{\Theta|X,\tilde{\Lambda}} \left[R(\Theta, X, \tilde{\Lambda}) \right] \right], \tag{1}$$

where,

- X denote the all the data,
- Λ all parameters, and
- Θ unobservable skill levels.

A feasible version of same minimization

- Start with a guess for parameter values, $\hat{\Lambda}^{(0)}$,
- Solve minimization by iterations starting with $s = 0$,

$$\hat{\Lambda}^{(s+1)} = \arg\min_{\tilde{\Lambda}} \mathbb{E}_X \left[\mathbb{E}_{\Theta|X,\hat{\Lambda}^{(s)}} \left[R(\Theta, X, \tilde{\Lambda}) \right] \right],$$

we have $\hat{\Lambda}^{(s)} \to \Lambda$ with large number of iterations.

EM Algorithm

Guess parameter values, $\hat{\Lambda}^{(0)}$ and start iteration with $s = 0$.

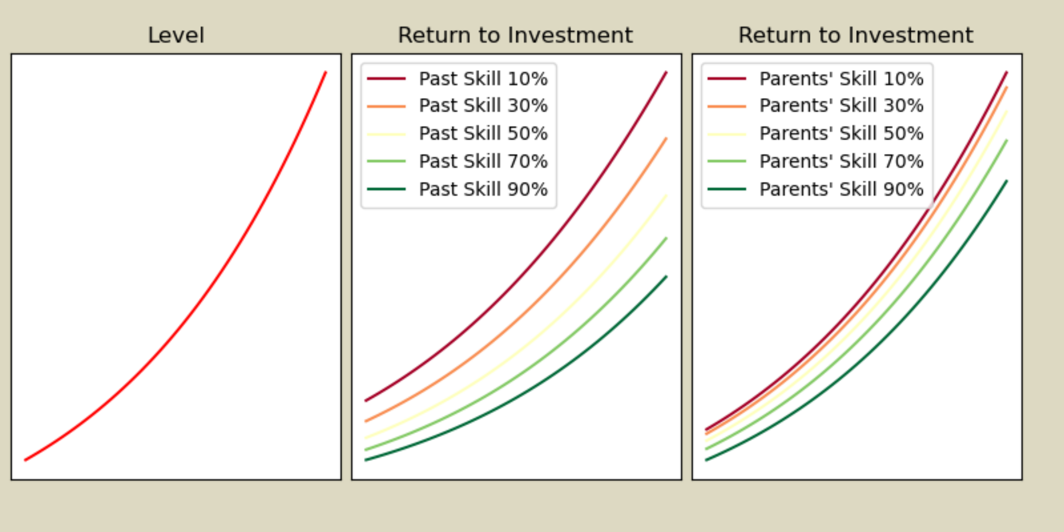
- Simulate underlying skill levels for each child i,

$$\theta_{im} \sim f(\theta \mid X_i, \hat{\Lambda}^{(s)}) \quad \text{for} \quad i = 0, \ldots, N, \quad m = 0, \ldots, M,$$

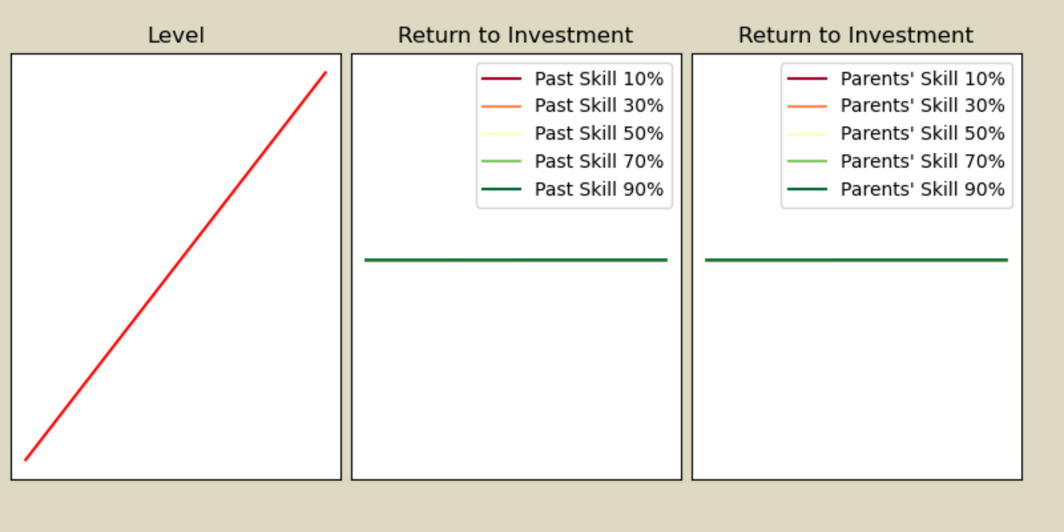
- Solve sample counterpart of minimization and update the parameters,

$$\hat{\Lambda}^{s+1} = \arg\min_{\tilde{\Lambda}} \frac{1}{N} \sum_{i=0}^{N} \frac{1}{M} \sum_{m=0}^{M} R(\theta_i, X_i, \tilde{\Lambda}).$$

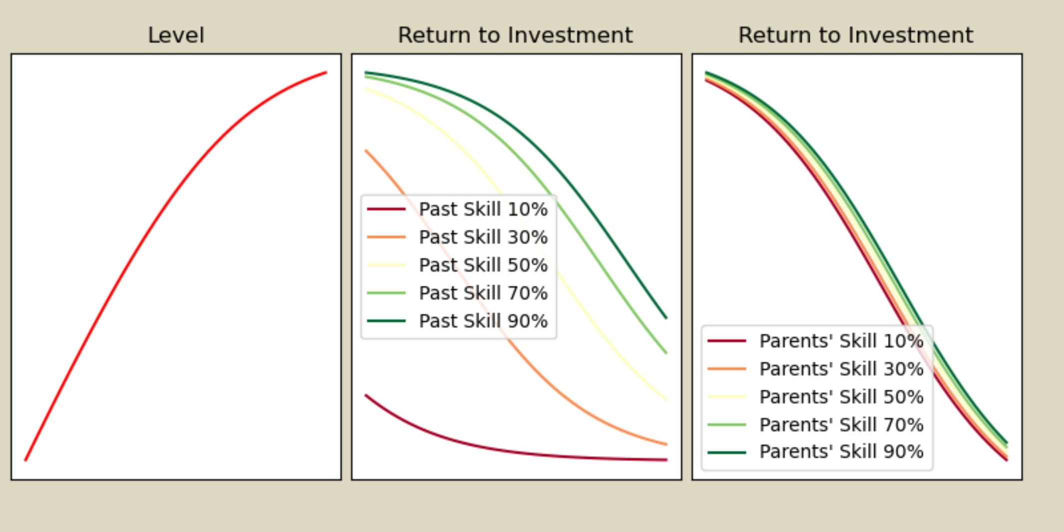
Baseline CES: Inputs are Substitutes



Cobb-Douglas: $\phi = 0$



Alternative CES: Inputs are Complements



Estimation Results for Aggregation Functions

Parental Education:

$$\ln\theta_P = \ln\theta_{mother} + 0.81 \times \ln\theta_{father} + 0.48 \times \ln\theta_{mother}\ln\theta_{father}$$

with 90% confidence intervals: (0.33, 2.02) and (0.13, 1.85).

Investment:

$$\ln I_j = \ln t_j^{mother} + 0.76 \times \ln t_j^{father} + 1.8 \times \ln m_j$$

with 90% confidence intervals: (0.30, 1.97) and (1.02, 3.17).

No Upward Trend in Intergenerational Rank-Rank Correlation